A SURFACTANT FILM SPREADING REGIME

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The self-similar problem of spreading of thin films of surfactants is examined with consideration of the effect of the total quantity of material present on the spreading process.

Interest has recently increased in study of the mechanisms whereby oil spills spread over sea and ocean surfaces [1-9]. In the later stages of this process, when the petroleum film thickness becomes sufficiently small, the main forces determining the growth of its horizontal dimensions are surface tension and viscosity. In this case the flow characteristics do not depend on the total quantity of spreading substance nor its surface concentration distribution [1, 2, 9]. However, in the final stages of the spreading process the film becomes so thin that it is necessary to consider the effect of surface concentration distribution of the material on the process. Similar problems occur in the study of spreading of a surfactant substance in the case where the total quantity of material is small and the surface tension regime sets in very soon.

Thus, we shall examine spreading of a film in a regime wherein it is necessary to consider the total quantity of surfactant present Q, initially located upon the surface of a viscous incompressible liquid.

From the condition of conservation of total quantity of material present, in the axisymmetric case we may write

$$2\pi \int_0^\infty \Gamma(r, t)\, r dr = Q. \tag{1}$$

Neglecting diffusion processes, the equation of conservation of quantity of material within the film may be written in the form [10]

$$\frac{\partial \Gamma}{\partial t} + \frac{1}{r}\frac{\partial (r\Gamma u)}{\partial r} = 0, \tag{2}$$

where u is the horizontal liquid velocity at the boundary with the film, directed along the radius.

We assume that surface tension over the film changes from the surface tension of the pure liquid σ_0 to the current σ value which is dependent on concentration, i.e., $\sigma = \sigma(\Gamma)$.

The resultant force produced by the surface tension gradient is compensated by a shear force τ, acting on the film from the direction of the liquid:

$$\nabla\sigma + \vec{\tau} = 0. \tag{3}$$

Using the results of laminar boundary-layer theory [10] with the assumption that the velocity u is constant over film thickness, from Eq. (3) we obtain

$$u = bK(\partial\Gamma/\partial r)^{2/3} r^{1/3}, \tag{4}$$

where $K = (\varkappa^2\nu/\mu^2)^{1/3}$; $\varkappa = \partial\sigma/\partial\Gamma$ is taken constant [11]; and b is a constant equal to 2.0857.

Substituting Eq. (4) in Eq. (2) we obtain

$$\frac{\partial \Gamma}{\partial t} + \frac{bK}{r}\frac{\partial [r^{4/3}\Gamma(\partial\Gamma/\partial r)^{2/3}]}{\partial r} = 0. \tag{5}$$

In the present case we are interested in the asymptotic stage of fluid flow where the film radius R significantly exceeds the initial radius R_0. In this stage the details of the

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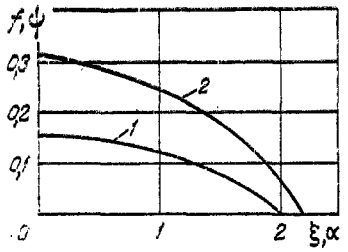

Fig. 1. Self-similar concentration function distribution.

initial concentration distribution are insignificant. To describe the process we represent the initial distribution in the form of a concentrated source

$$\Gamma(r,\ t_1) = 0 \quad \text{at} \quad r \neq 0, \tag{6}$$

where t_1 is the arbitrary commencement time of this stage.

Thus, the problem reduces to solution of Eq. (5) with initial condition (6) in the presence of conservation law (1). The solution will depend on the parameters K, Q, $(t - t_1)$. Using considerations of dimensionality, we will seek a solution of Eq. (5) in the form

$$\Gamma = [KQ^{-2/3}\,(t-t_1)]^{-3/4}\,f(\xi), \tag{7}$$

where

$$\xi = [KQ^{2/3}\,(t-t_1)]^{-3/8}\,r. \tag{8}$$

Substituting Eq. (7) in Eq. (5) and integrating the ordinary differential equation thus obtained for the function $f(\xi)$, we find

$$f(\xi) = \begin{cases} \gamma\,(\xi_0^2 - \xi^2)/2, & 0 \leqslant \xi < \xi_0, \\ 0 & \xi_0 < \xi, \end{cases} \tag{9}$$

where $\gamma = (3/8b)^{3/2}$; ξ_0 is a constant, defined by conservation law (1); $\xi_0 = (4/\gamma\pi)^{1/4} \approx 2$. A graph of the function $f(\xi)$ is shown in Fig. 1 (curve 1).

From Eq. (8) we obtain an expression for propagation of the forward front

$$R(t) = 2.0\,[KQ^{2/3}\,(t-t_1)]^{3/8}. \tag{10}$$

Solving the problem for the planar case in an analogous manner, we obtain

$$\Gamma = [KQ_1^{-4/3}\,(t-t_1)]^{-1/2}\,\psi(\alpha),$$

$$\alpha = [KQ_1^{2/3}\,(t-t_1)]^{-1/2}\,x,$$

$$\psi(\alpha) = \begin{cases} \gamma_1\,(\alpha_0^2 - \alpha^2)/2, & 0 \leqslant \alpha < \alpha_0, \\ 0, & \alpha_0 < \alpha, \end{cases} \tag{11}$$

where $\gamma_1 = (1/2b)^{3/2}$; $\alpha_0 = (3/2\gamma_1)^{1/3} \approx 2{,}3$; Q_1 is the total quantity of spreading material. Then we find the expression for propagation of the forward front

$$L(t) = 2{,}3\,[KQ_1^{2/3}\,(t-t_1)]^{1/2}. \tag{12}$$

A graph of the function $\psi(\alpha)$ is shown in Fig. 1 (curve 2).

Thus, a self-similar solution has been found to the problem of spreading of a surfactant in the case where the total amount of material present must be considered. Both planar and spatial cases have been considered.

From analysis of the results obtained above, it follows that the surfactant substance propagates from the source at a finite rate in a manner such that a clear boundary exists between the region occupied by the material and the bare surface. The film spreading has the character of a wave, the front of which is this boundary.

Similar solutions in the form of a thermal wave in cases of nonlinear thermal conductivity were first studied in [12]. Among the studies of this problem, we will note [13, 14], which considered the halting of the thermal wave front (metastable heat localization). Simi-

lar phenomena may occur in the development of a turbulized layer in a homogenous liquid [15]. A finite propagation front velocity was also observed in a study of the viscous stage of spreading of spots of mixed fluid in stably stratified media [16].

We note that consideration of the effect of surface concentration distribution inhomogeneity of the material within the film upon the final stage of the spreading process leads to a reduction in the forward front propagation rate as compared to the "surface tension" regime, with both L and R being proportional to $t^{3/4}$ [1, 9]. A definite analog of this case of film spreading is the two-stage diffusion process in solutions near the critical point, where there is initially a finite diffusion delay time, with a rapid change in speed of the process developing later [17].

NOTATION

Γ, Q, surface concentration and total quantity of surfactant; σ_0, σ, surface tensions of pure liquid and film-coated liquid; $\vec{\tau}$, shear stress; μ, ν, dynamic and kinematic viscosity coefficients; $\varkappa=\partial\sigma/\partial\Gamma$, parameter characterizing change in surface tension with concentration; x, r, horizontal coordinates in planar and axisymmetric cases; t, current time; t_1, arbitrary time of given regime commencement; L, R, maximum horizontal film dimensions in planar and axisymmetric cases; α, ξ, $\psi(\alpha)$, $f(\xi)$, self-similar variables and concentration functions in planar and axisymmetric cases; b, γ, γ_1, constants.

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